

Homework 2

Psychology 310

Instructions. Answer the following questions. Show your R code, your input, and your output. Feel free to email me for hints if you get stumped.

- (10 points) For each of the following expressions, for variables X and Y and constants a, b, c, d tell (i) whether or not the expression is a linear combination of X and Y , and (ii) if the expression is a linear combination, what are the linear weights for X and for Y .
 - $(9X - 6Y)/3$
 - $(X + Y)^2$
 - $4(X + Y)$
 - $a^2X + b^2Y$
 - $2XY - 5X$
- (5 points) X has a mean of 10, Y has a mean of 20. What is the mean of the linear combination $X + 4Y$?
- (10 points) X has a variance of 36, Y has a variance of 49. If X and Y have a correlation of .50, what is the variance of $3X - Y$?
- (10 points) Given the same facts about X and Y as the previous problem, what is the covariance between the two linear combinations W and M given by the formulas $W = X + Y$ and $M = 2X + 3Y$?
- (10 points) Here are some simple set theory problems. Suppose $\Omega = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2\}$, $B = \{1, 2, 3\}$, and $C = \{1, 4, 5, 6\}$ Find the following sets: (*Hint:* Load the R library `arules` and study the `union`, `intersect`, and `setdiff` functions. You can easily write your own `complement` function. Then use R to solve the problems.)
 - $B - A$
 - $A \cup B$
 - $(A \cup B) - C$
 - $A \cap C$
 - $\overline{A \cup B}$

- (f) $\overline{A \cap B}$
- (g) $(A \cup B) - (A \cap C)$
- (h) $C - A$
- (i) $C \cup A$
- (j) $A - C$

6. (5 points) The sample mean \overline{X}_\bullet is a linear combination of the N observations, which are generally assumed to be independent and hence uncorrelated, and which are assumed to each have the same mean μ and variance σ^2 . Suppose a population has a normal distribution, a mean μ of 100 and a standard deviation σ of 15. If you take samples of size $N = 25$ from this population, and compute the sample means \overline{X}_\bullet , they will, over repeated samples, have a distribution known as the sampling distribution of the sample mean. What will be the mean and variance of this sampling distribution? *Hint.* Suppose N is 2 instead of 25, and apply the linear combination theory we covered in class. Then generalize to $N = 25$.
7. (5 points) Given the same information as the preceding problem, what is the probability that a sample mean \overline{X}_\bullet based on a random sample of size 25 will be greater than 108?
8. (10 points) Given the following data, find the linear regression equation for predicting Y from X , and then compute the predicted value of Y when $X = 4$.

X	Y
5	7
4	11
8	14
7	11
2	5
3	6
9	16
11	22
10	18
6	13

9. (10 points) Let C be the event you pass the course. Let E_1 be the event you pass exam 1. The probability that you pass the course, given that

you pass the first exam, is .90. The probability that you pass the first exam is also .90. What is the probability that you pass the first exam and pass the course, i.e., $\Pr(E_1 \cap C)$?

10. (10 points) The probability that you are a Glog is .70. The probability that you are a Zorg is .70. The probability that you are a Glog or a Zorg or both is .80. What is the probability that you are a Glog *and* a Zorg?
11. (5 points) What is the probability of throwing 5 independent fair dice and having them all come up sixes?
12. (5 points) Suppose the probability that a newborn child will be a boy is .5124, and that the probability the child will be a girl is .4876. Anna decides to keep having children until she has her first girl, at which point she will stop. Discounting the possibility of more than one child being born at a time, what is the probability that she will have 2 boys and one girl?
13. (5 points) In the game of Bridge, 13 card hands are dealt. When you pick up your hand of 13 cards, what is the probability that they are all black (i.e., all spades or clubs)?