Homework 3

Psychology 310

Instructions. Answer the following questions. Show your R code, your input, and your output. Feel free to email me for hints if you get stumped.

1. (10 points). In Lab 4, I stated that, to obtain a continuous uniform random variable with mean μ and standard deviation σ, you specify a $\mathrm{U}(\mu - \sqrt{3\sigma^2}, \mu + \sqrt{3\sigma^2})$ random variable, that is, a uniform random variable with limits of $\mu - \sqrt{3\sigma^2}, \mu + \sqrt{3\sigma^2}$. Prove this result, using the fact that a uniform random variable with limits A and B has mean $(B + A)/2$ and variance $(B - A)^2/12$.

2. (5 points). Suppose that, in the long run, the Bears are a .600 team, in the sense that if they played an infinite number of games, they would win 60% of them. Furthermore, assume that the Bears performance over a 16 game season can be approximated by a binomial distribution with $N = 16$ and $p = .60$. Suppose that the Bears need to win 10 or more games this season to make the playoffs. What is the probability that the Bears will make the playoffs?

3. (5 points). You have a friend who believes in ESP. You decide to test your friend. You throw a fair die 20 times and after each throw, your friend, sitting in another room, tries to guess the number you rolled. Your friend guesses 9 right out of 20. What is the probability that a person guessing randomly would get 9 *or more* right?

4. (5 points). You awaken from a deep slumber, and realize abruptly that this is the morning of your physics test. The problem is, you haven't attended a single class all semester, and you do not now, nor did you ever, know anything about physics. You go to the classroom and realize to your delight that the test consists of 20 true-false questions. You need a 60% grade to pass the course. Assuming that your responses are completely random, and so the probability of correctly answering a question is 0.50, what is the probability that you will pass the course?

5. (5 points). In baseball, two teams play a best-of-seven format in the World Series. Suppose that World Series games are independent binomial trials, and the better team has a long run probability of success of .55 against the inferior participant. What is the probability that

the better team wins? (Note: The series stops when either team has won 4 games. So the length of a World Series varies.)

6. (30 points). In the early days of baseball, the only "playoffs" were the World Series, a best-of-7 playoff between the winner of the American League and National League pennants. Moreover, there was no inter-league play other than the World Series.

   (a) (10) Suppose that, in the old fashioned system, the best team in baseball, team A, in terms of long run probability of winning a game, was in the American League and had a long run probability of winning a game of .650 when playing anyone other than the second best team in the American League, team B. Team B had a long run probability of winning a game of .620 when they are not playing anyone other than the best team. When the two best teams play each other, Team A has a long run probability of winning of .530.

   As a simplifying assumption, suppose that games are independent binomial trials, and these two teams were to play 154 game schedules of which 22 games are against each other and 132 are against other teams. Use simulation of 100,000 seasons to estimate the distribution of the difference in the number of wins.

   What is the probability that the best team will win fewer games than the second-best team in a 154 game season? (Hint. If you have the difference in the number of wins in a variable called `diff`, this probability is estimated by `mean(diff < 0)`).

   (b) (5) What is the probability that the two teams will win *exactly the same number of games*?

   (c) (5) Assume that there is no other team with a non-negligible chance of winning the pennant, and that if the two best teams tie, they play one game, with the winner taking the pennant. What is the probability that the best team in the American league will end up winning the pennant?

   (d) (10) Now, assume that exactly the same situation holds in the National League. So each team is going to send its first or second best team to the World Series, and at that point the two teams play 7 games, with the first team to win 4 games taking the championship. (Note, they only play until one team wins 4 games. They do not play 7 games each year!) Further more,

again assuming individual games are independent binomial trials, the probability of win in any game is as follows:

- American League Best wins a game against National League Best — .54
- American League Best wins a game against National League Second Best – .59
- American League Second-Best wins a game against National League Best – .49
- American League Second-Best wins a game against National League Second Best – .54

What is the probability that the best team in baseball, i.e., American League Team A, ends up as World Series champion? (Use your simulation probabilities from the previous question in combination with the 4 probabilities listed above to help produce your answer. If you wish, test out your answer with simulation!)

7. (10 points) In class, we discussed the concepts of one-sided (or one-tailed) vs. two-sided (or two-tailed) hypothesis tests. We also discussed the distinction between Reject-Support testing and Accept-Support testing. Drawing on these ideas, answer the following questions.

   (a) (5) An experimenter believes there is a positive difference between an experimental treatment population and a control population. That is, he believes that $\delta = \mu_{\text{experimental}} - \mu_{\text{control}} > 0$. He sets up a completely randomized 2-group experiment. What is his statistical null hypothesis? Is it one-sided or two-sided?

   (b) (5) An experimenter believes that a drug will affect behavior on a particular cognitive test, but he has no idea whether it will improve performance or hurt performance on the test. He sets up a randomized, two group experiment. What is the statistical null hypothesis? Is the hypothesis one-sided or two-sided?

8. (10 points) Consider the sample mean based on N independent observations. Briefly describe known facts about the distribution of the sample mean $\overline{X}_\bullet$ (for example, the formulas for the expected value and variance of its sampling distribution) that would allow you to conclude that, as an estimator of μ, $\overline{X}_\bullet$ is

   (a) (5) Unbiased

(b) (5) Consistent

9. (20 points). Suppose you plan to test a drug on groups of male and female rats. You want to compare the effect of the drug on males to the effect on females Your actual belief is that the drug effect, measured as a mean difference between experimentals and controls, will be different for males and females.

   (a) (5) What is the statistical null hypothesis in this situation? Is it one-sided or two-sided?

   (b) (10) Construct a 4-sample t-statistic to test this hypothesis. Give the formula.

   (c) (5) Construct a formula for a confidence interval on the quantity $\kappa = \mu_1 - \mu_2 - \mu_3 + \mu_4$.