

The Normal Distribution

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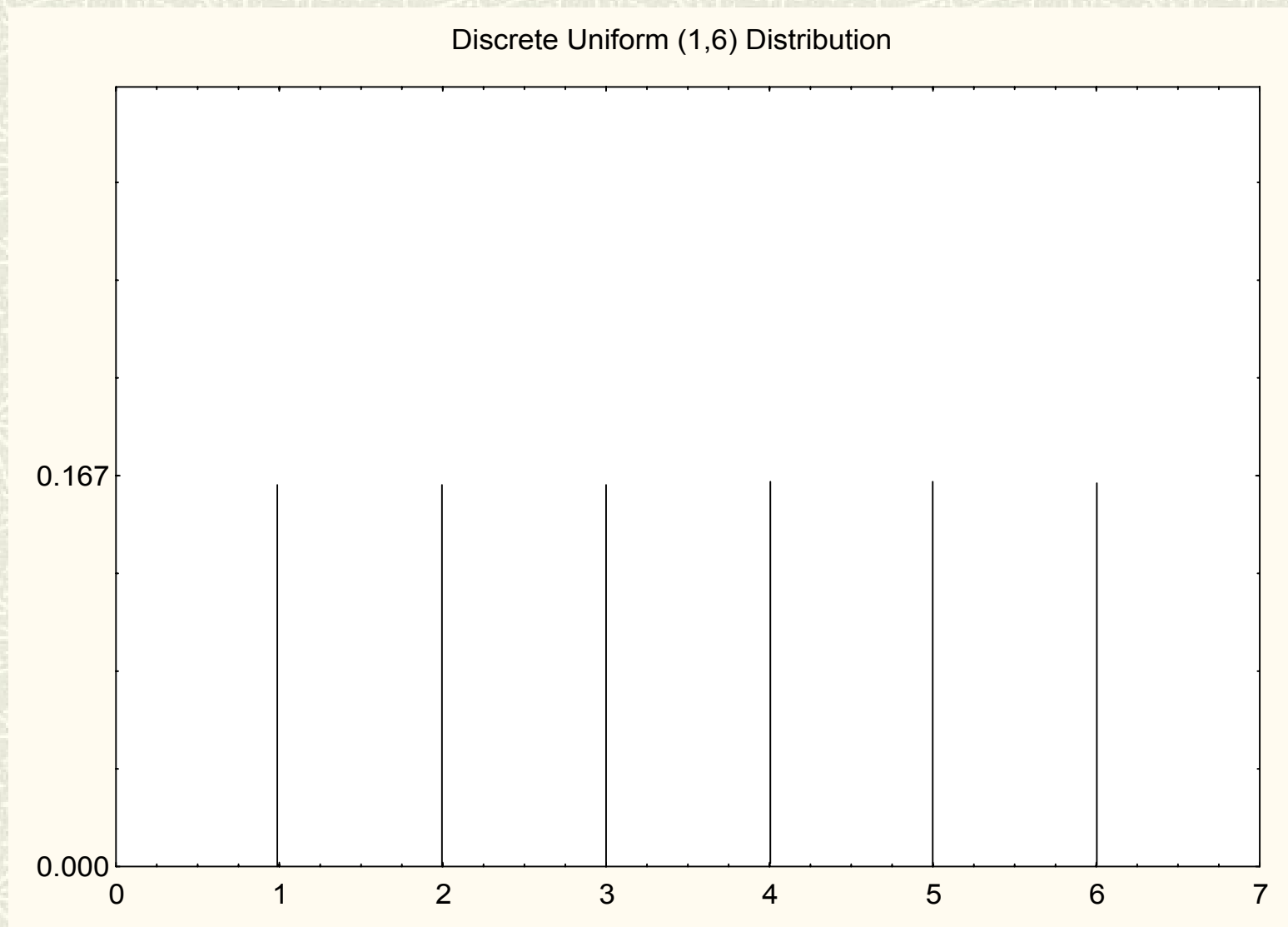
Types of Probability Distributions

- # There are two fundamental types of probability distributions
 - Discrete
 - Continuous
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Discrete Probability Distributions

- # These are used to model a situation where
 - The number of events (things that can happen) is *countable* (i.e., can be placed in correspondence with integers)
 - Probabilities sum to 1
 - Each event has a probability

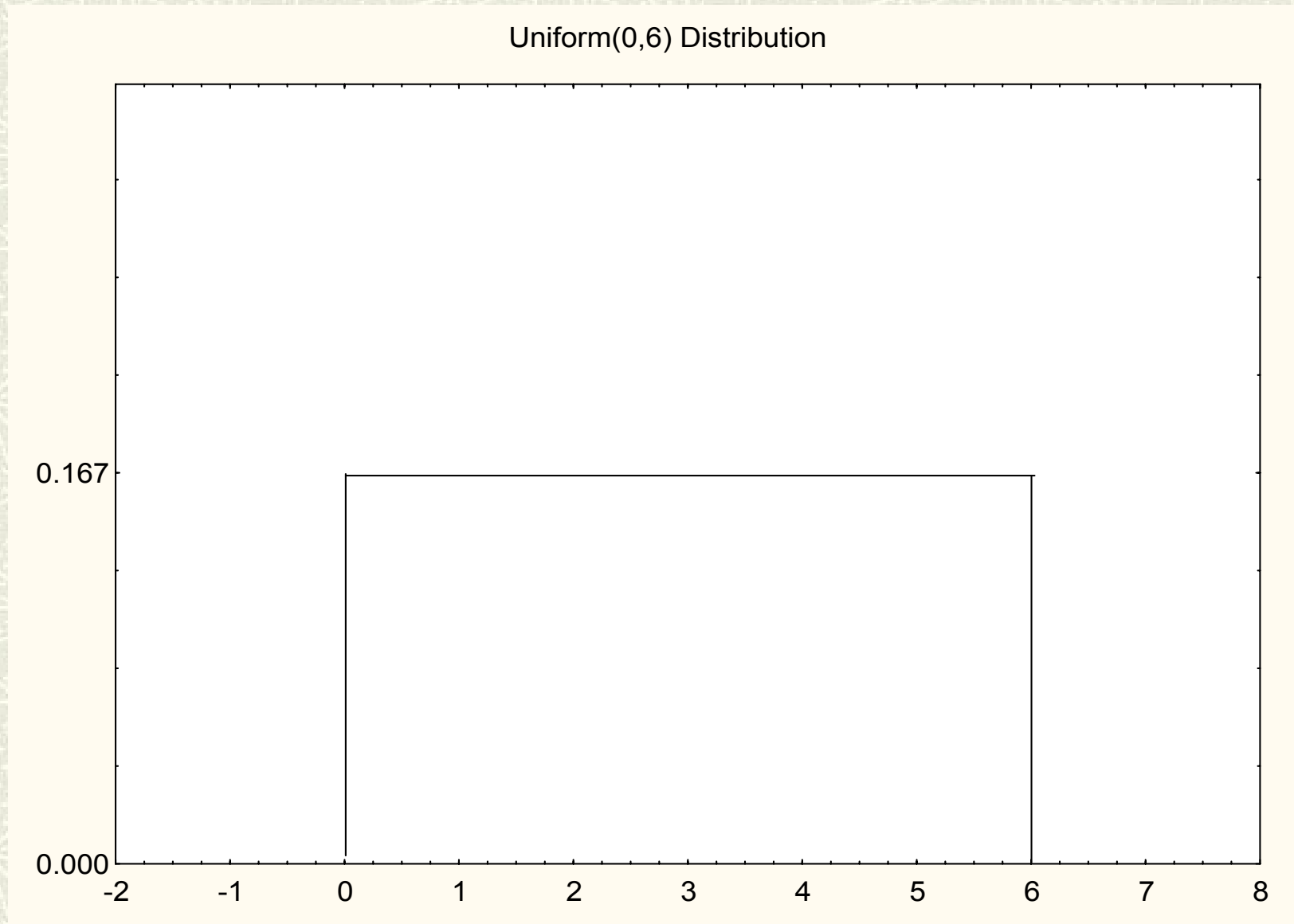
A Discrete Probability Function



Continuous Probability Distributions

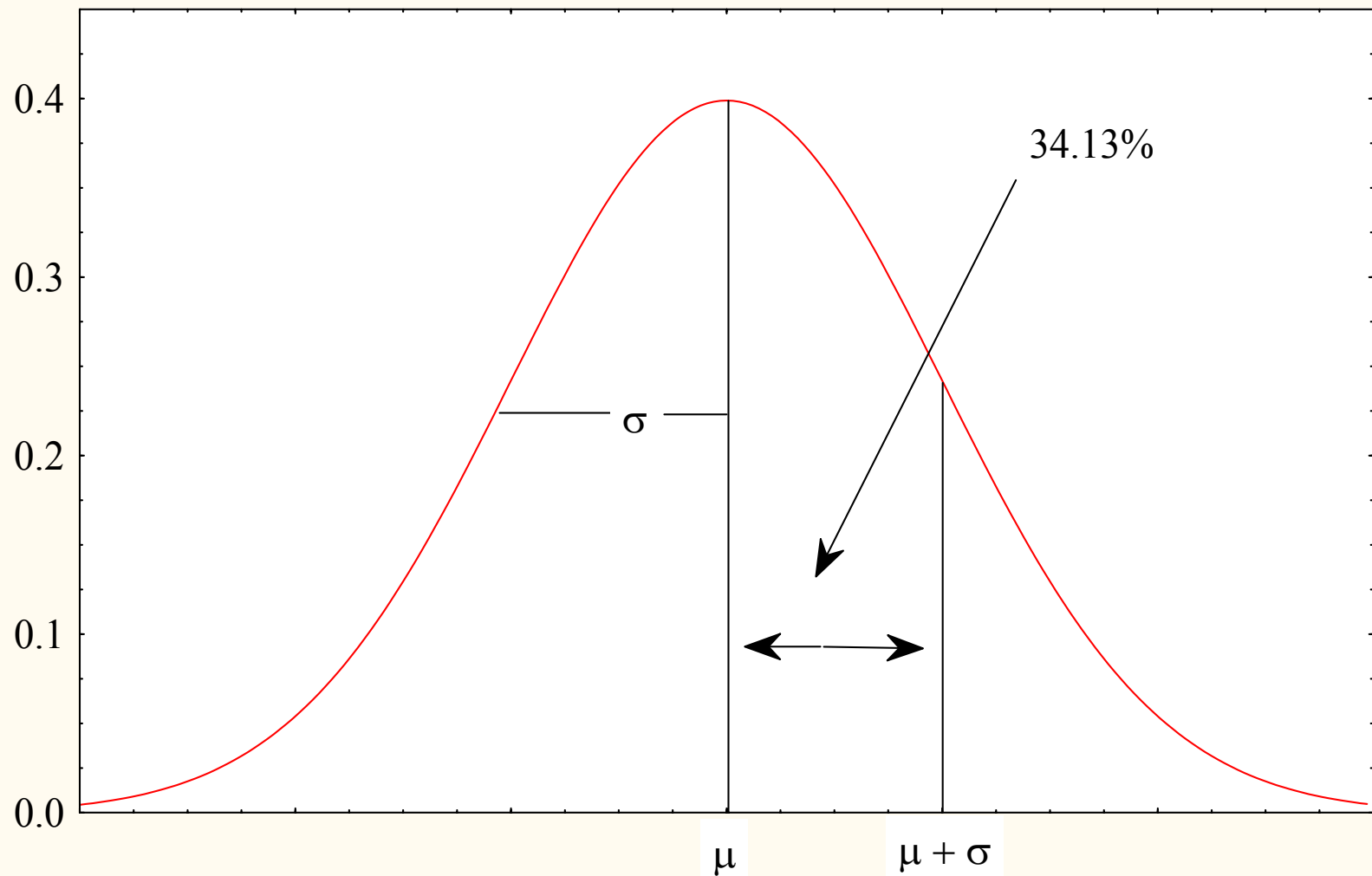
- # These are used to model situations where the number of things that can happen is not countable
 - # Probability of a particular outcome cannot be defined
 - # Interval probabilities can be defined: *Probability of an interval is the area under the probability density curve between the endpoints of the interval*
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Probability Density Function

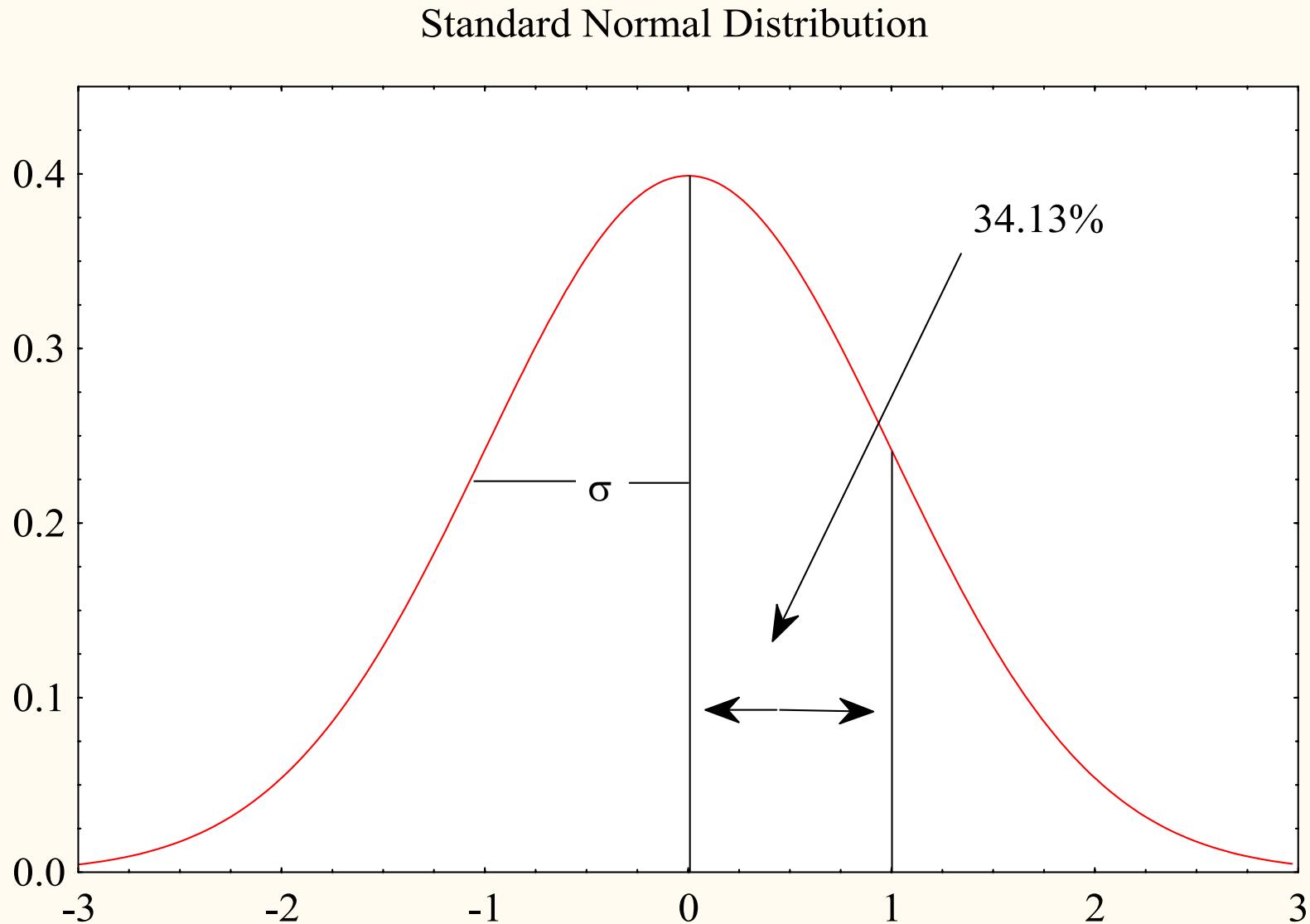


The Normal Distribution Family

Standard Normal Distribution



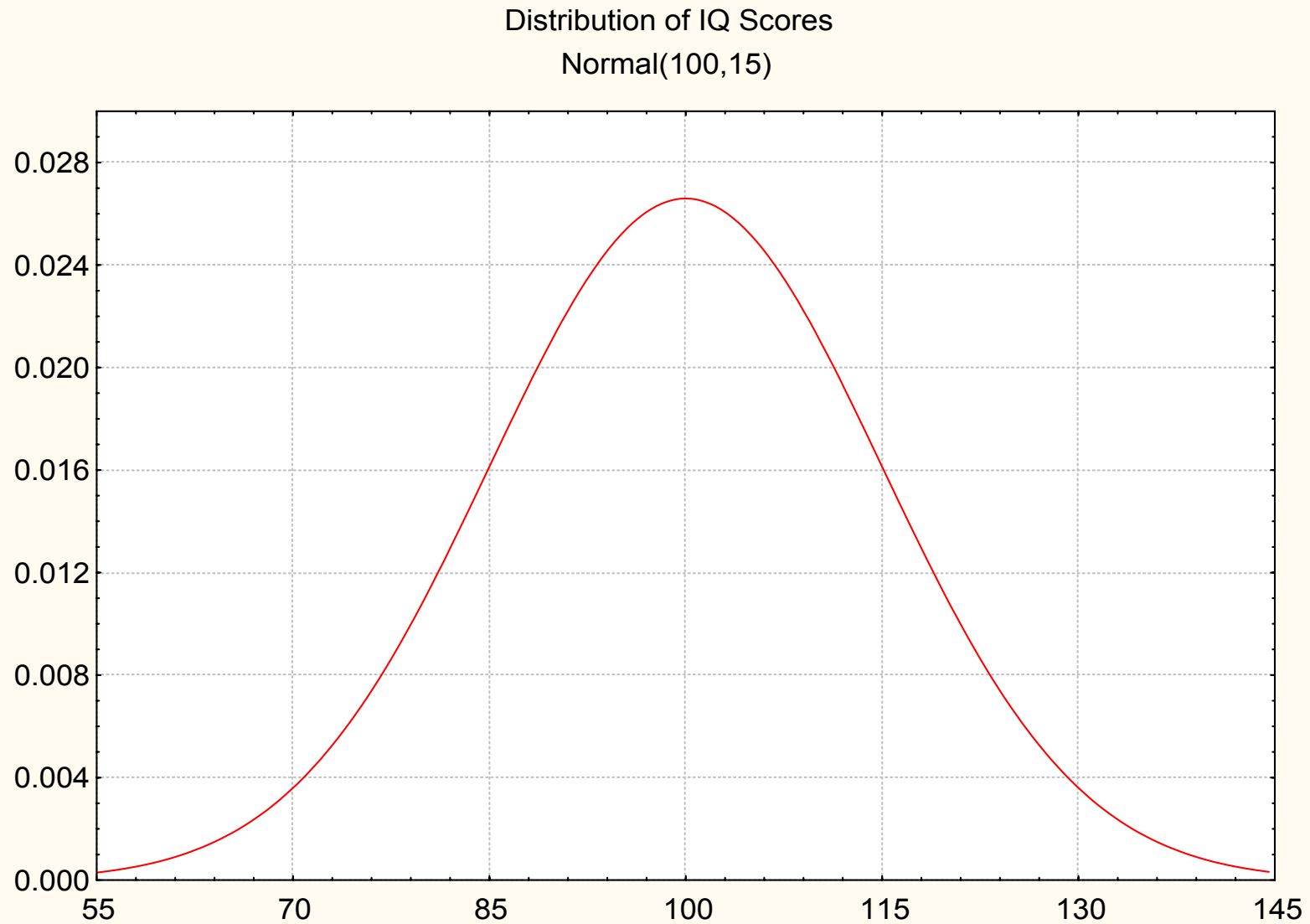
The Standard Normal Distribution



The Normal Curve Table

<u>Z</u>	<u>F(Z)</u>	<u>Z</u>	<u>F(Z)</u>
0.25	59.87	1.75	95.99
0.50	69.15	1.96	97.5
0.75	77.34	2.00	97.72
1.00	84.13	2.25	98.78
1.25	89.44	2.326	99.00
1.50	93.32	2.50	99.38
1.645	95.00	2.576	99.50

Some Typical “Routine” Problems



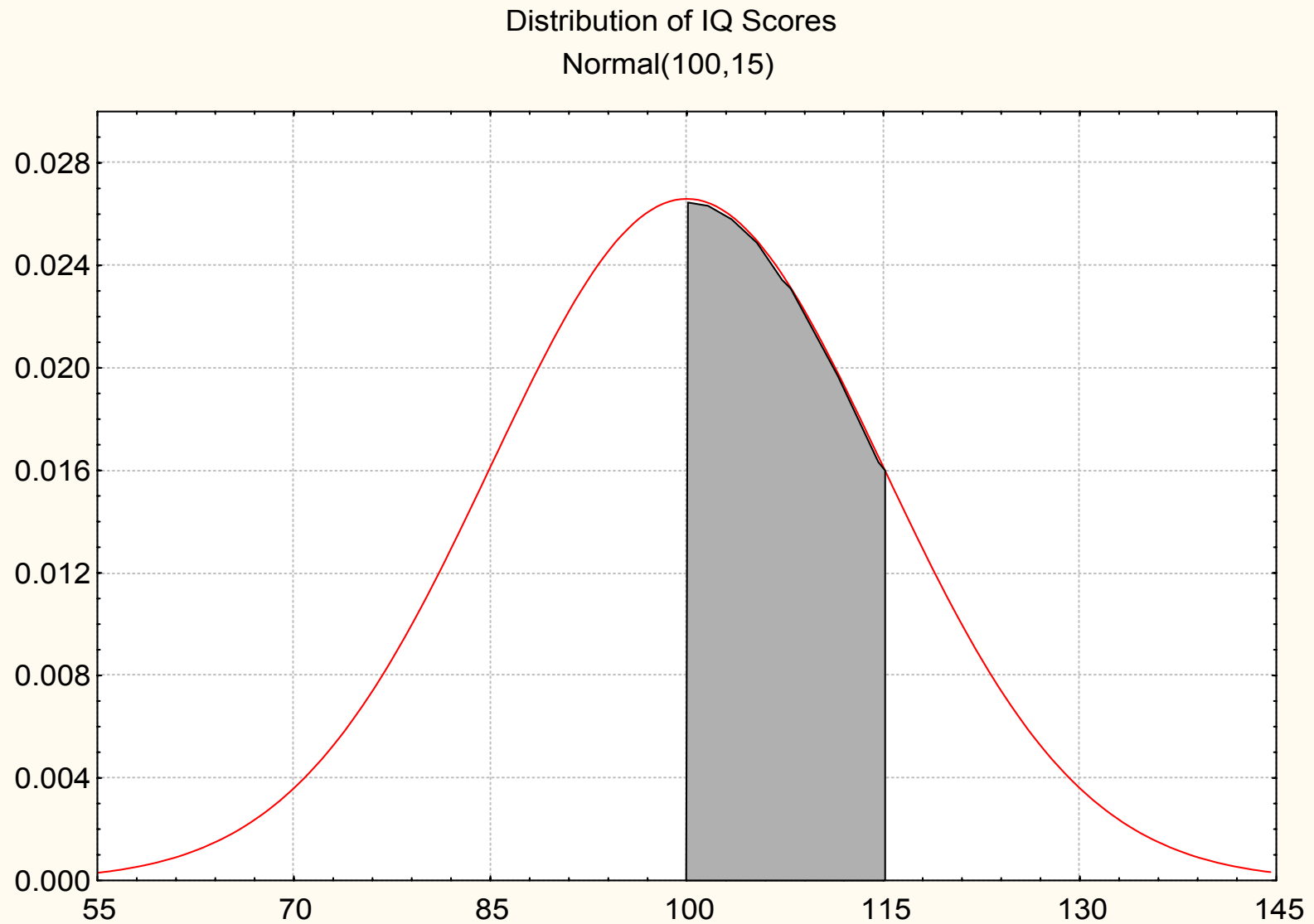
Some Typical “Routine” Problems

- # 1. What percentage of people have IQ scores between 100 and 115?
 - # 2. What percentage of people have IQ scores greater than 130?
 - # 3. What percentage of people have IQ scores between 70 and 85?
 - # 4. What is the percentile rank of an IQ score of 122.5?
 - # 5. What IQ score is at the 99th percentile?
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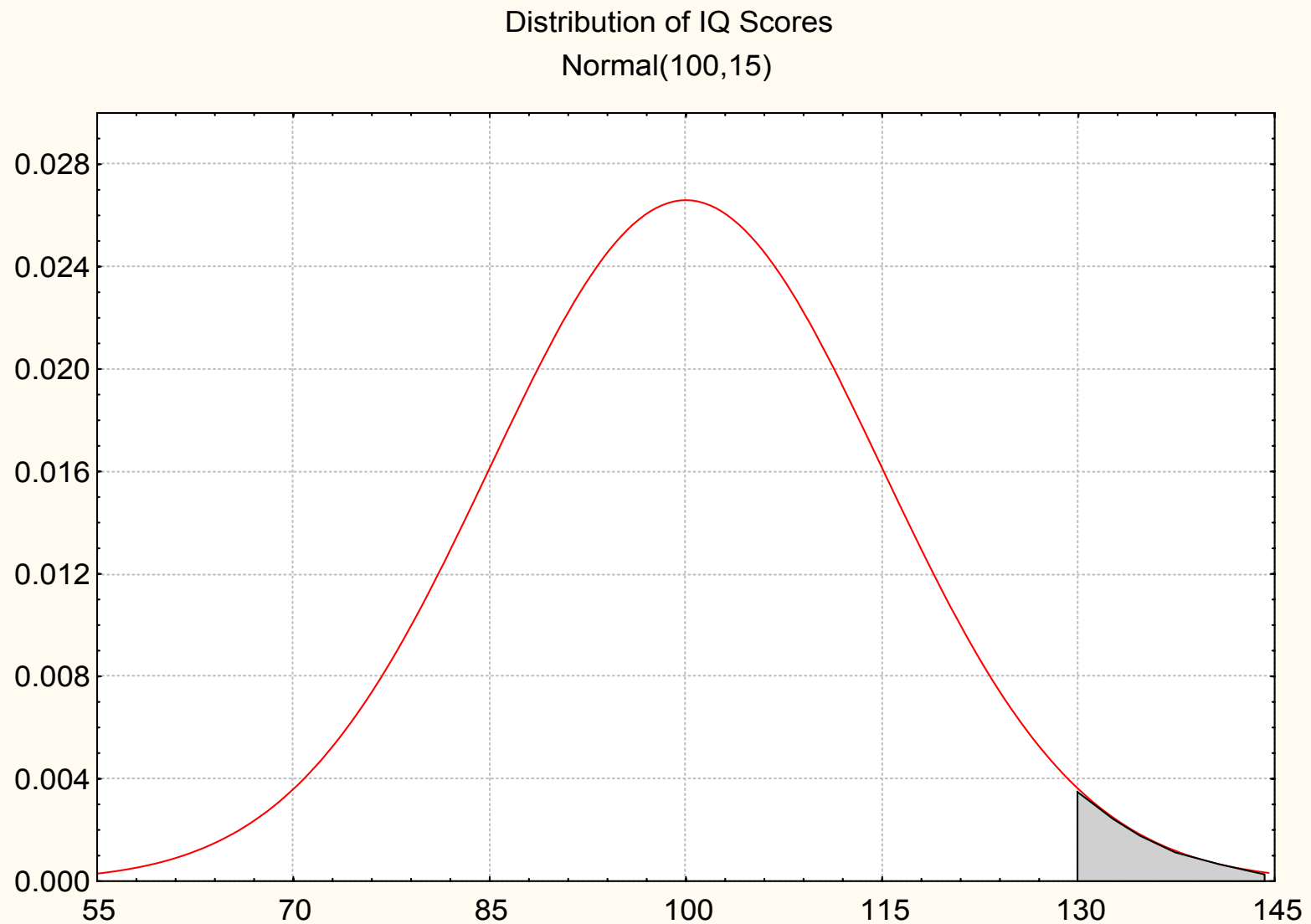
General Strategy for Normal Curve Problems

- # *ALWAYS* draw the picture
 - # Estimate the answer from the picture, remembering:
 - Symmetry
 - Total area is 1.0
 - Area to the left or right of center is .50
 - # Convert to Z -score form
 - # Compute interval probability by subtraction
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Problem #1



Problem #2



Problem #2 (ctd)

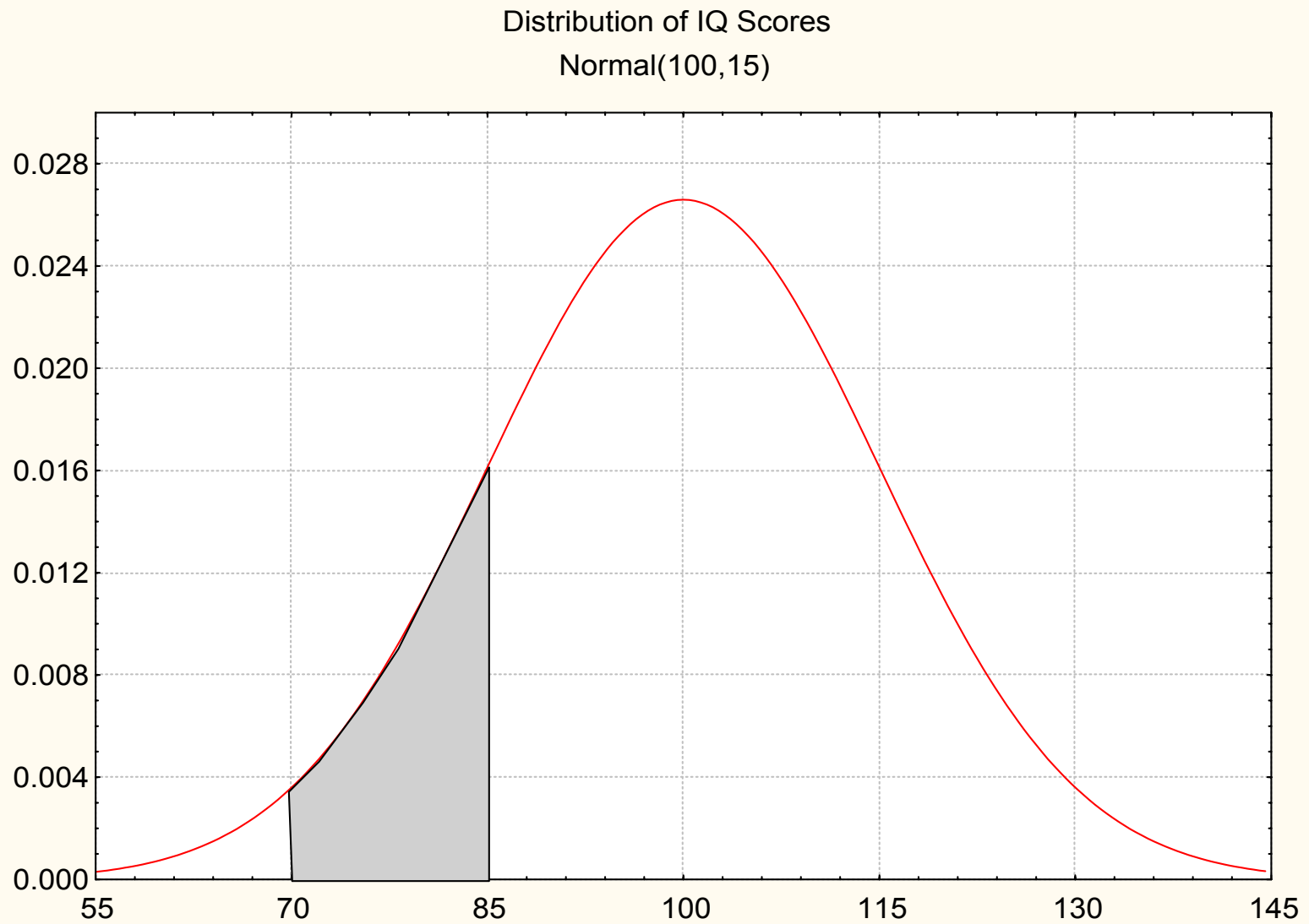
#

$$Z_{130} = \frac{130 - 100}{15} = +2.00$$

The Normal Curve Table

<u>Z</u>	<u>F(Z)</u>	<u>Z</u>	<u>F(Z)</u>
0.25	59.87	1.75	95.99
0.50	69.15	1.96	97.5
0.75	77.34	2.00	97.72
1.00	84.13	2.25	98.78
1.25	89.44	2.326	99.00
1.50	93.32	2.50	99.38
1.645	95.00	2.576	99.50

Problem #3



Problem #3

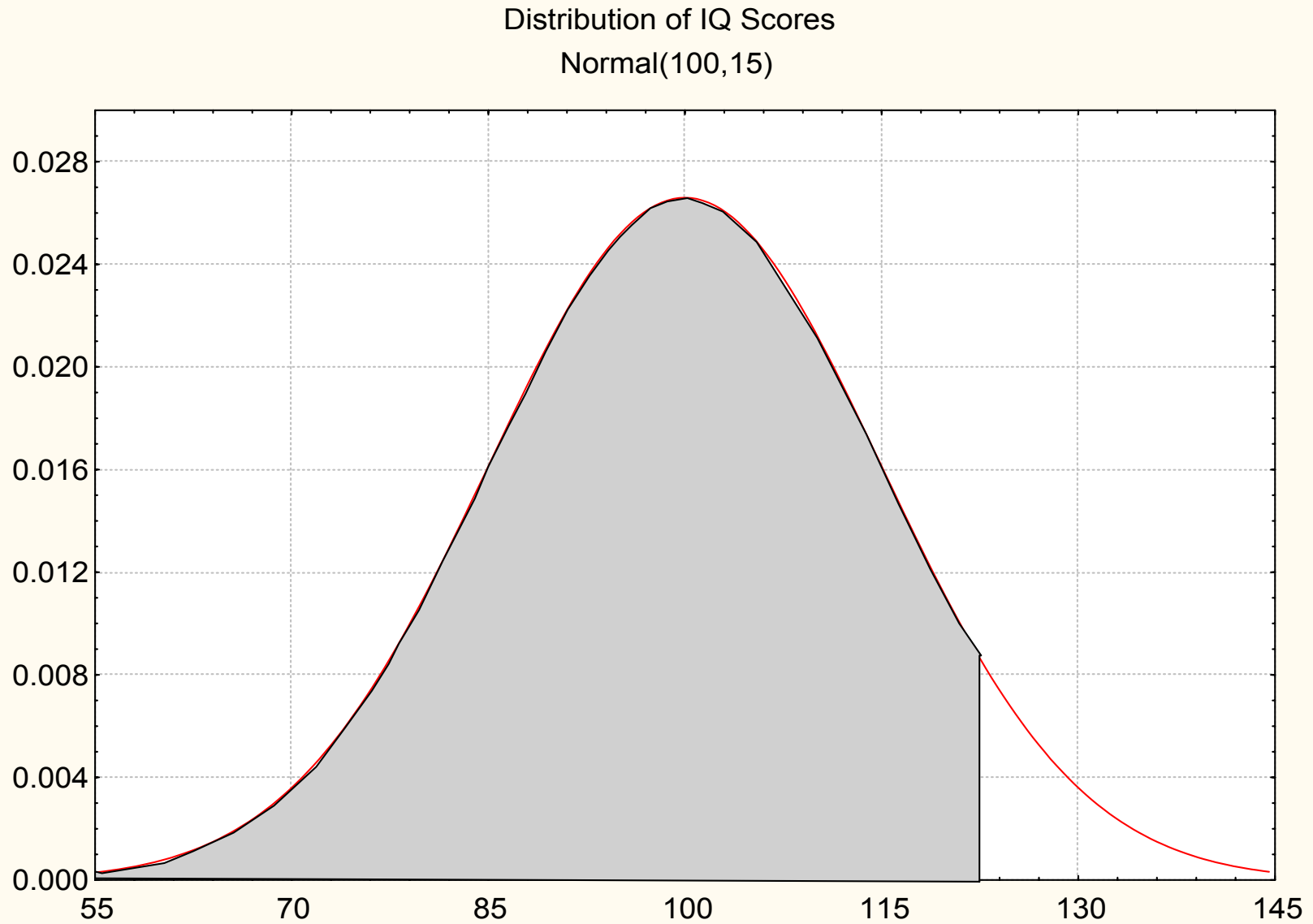
$$Z_{70} = \frac{70 - 100}{15} = -2.00$$

$$Z_{85} = \frac{85 - 100}{15} = -1.00$$

The Normal Curve Table

<u>Z</u>	<u>F(Z)</u>	<u>Z</u>	<u>F(Z)</u>
0.25	59.87	1.75	95.99
0.50	69.15	1.96	97.5
0.75	77.34	2.00	97.72
1.00	84.13	2.25	98.78
1.25	89.44	2.326	99.00
1.50	93.32	2.50	99.38
1.645	95.00	2.576	99.50

Problem #4



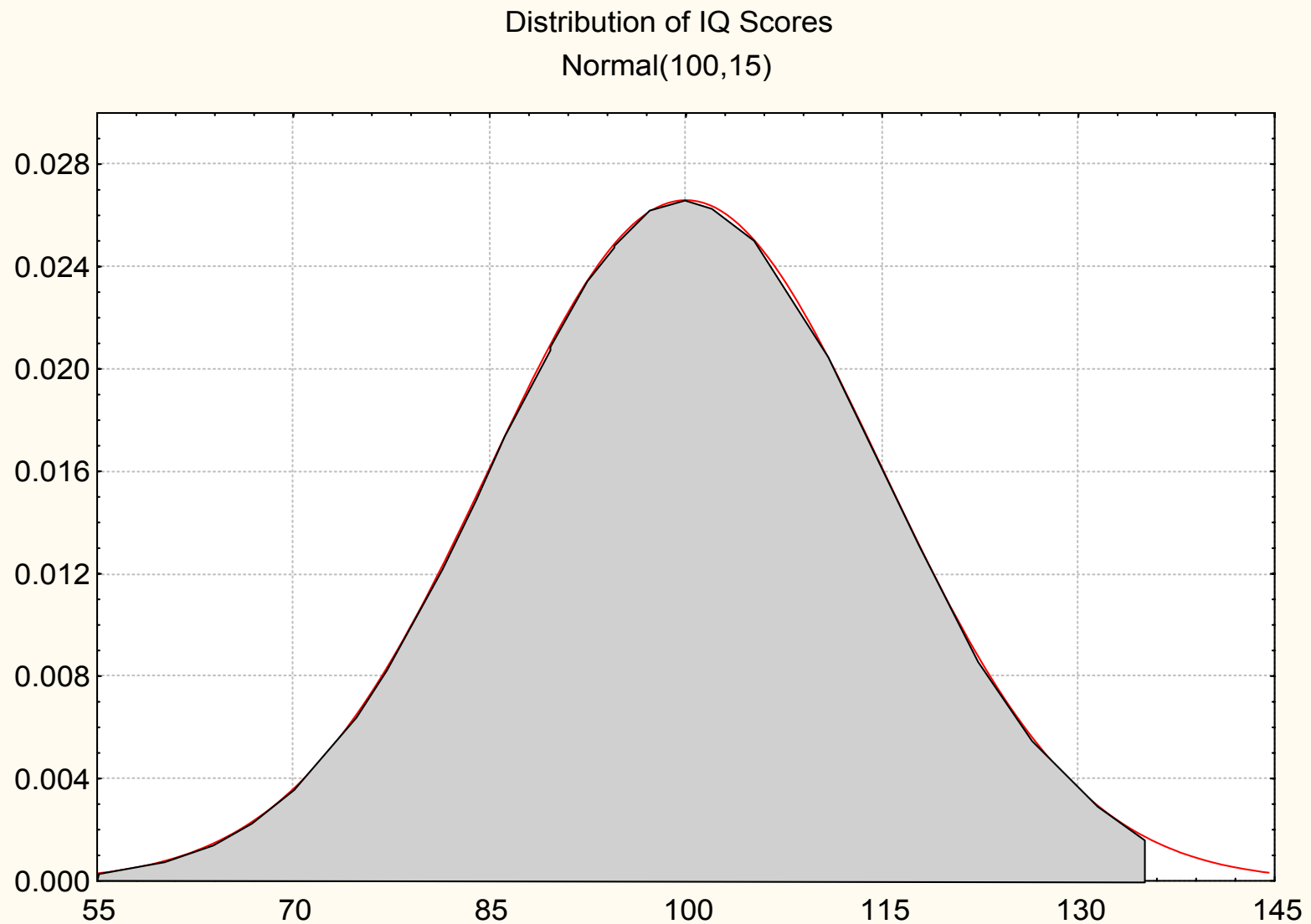
Problem #4

$$Z_{122.5} = \frac{122.5 - 100}{15} = 1.50$$

The Normal Curve Table

<u>Z</u>	<u>F(Z)</u>	<u>Z</u>	<u>F(Z)</u>
0.25	59.87	1.75	95.99
0.50	69.15	1.96	97.5
0.75	77.34	2.00	97.72
1.00	84.13	2.25	98.78
1.25	89.44	2.326	99.00
1.50	93.32	2.50	99.38
1.645	95.00	2.576	99.50

Problem #5



Problem #5

- # Find Z-score corresponding to 99th percentile.
 - # Convert to a raw score.
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Relative Tail Probability Problems

- # These problems involve analyzing *relative probabilities of extreme events in two populations*. Such problems:
 - Are seldom found in textbooks
 - Are often relevant to real world problems
 - Sometimes produce results that are counterintuitive
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Relative Tail Probability #1

- # Group A has a mean of 100 and a standard deviation of 15. Group B has a mean of 100 and a standard deviation of 17. What is the *relative likelihood* of finding a person with a score above 145 in Group B, relative to Group A?

The Normal Curve Table

<u>Z</u>	<u>Area Above Z</u>
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2.6	.0047
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2.65	.00405
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2.7	.0035
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3.0	.00135
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Answer: About 3 to 1.

Relative Tail Probability #2

- # Group A has a mean of 100 and a standard deviation of 15. Group B has a mean of 100 and a standard deviation of 17. What is the *relative likelihood* of finding a person with a score above 160 in Group B, relative to Group A?

The Normal Curve Table

<u>Z</u>	<u>Area Above Z</u>
3.529	.000208
4.00	.0000317

Answer: About 6.57 to 1.
