Psychology 312

Course Website: http://www.statpower.net

Course Information Sheet
Course Textbook

Immediately: (a) Read First 5 pages of Matrix Algebra Handout. Note where the "sticking points" are for you. (b) Download *R* and install it.

Introduction to Multivariate Analysis

What is a multivariate procedure?

A procedure is *multivariate* if it involves several variables, and *univariate* if it involves only one variable. Of course, this distinction is frequently blurred in practice.

Some Typical Multivariate Analyses

Data Modeling and Prediction

Linear and Nonlinear Regression

Understanding Group Differences

Multivariate Analysis of Variance (MANOVA)
Discriminant Analysis

Understanding Group Commonalities

Canonical Correlation and Redundancy Analysis

Data Structure, Data Reduction, and Beyond

Component Analysis
Exploratory Factor Analysis
Confirmatory Factor Analysis
Structural Equation Modeling

Mathematical Preliminaries

Hypothesis Testing Logic

Algebra of Variances and Covariances

- ***Linear Transformations***
- ***Linear Combinations***

t Tests

ANOVA

Matrix Algebra

Hypothesis Testing Logic

Type I Error
Type II Error
Power
Accept-Support Testing
Reject-Support Testing
Equivalence Testing

Algebra of Variances and Covariances

Linear Transformations

$$Y_i = aX_i + b \tag{1.1}$$

Linear transformation rule for means

$$\overline{Y}_{\bullet} = a\overline{X}_{\bullet} + b \tag{1.2}$$

Linear transformation rule for variances

$$S_Y^2 = a^2 S_X^2 \qquad (1.3)$$

Linear transformation rule for standard deviations

$$S_Y = |a| S_X \qquad (1.4)$$

Linear combinations

$$Y_i = \sum_{j=1}^{J} c_j X_{ij} \qquad (1.5)$$

Mean rule for linear combinations

$$\overline{Y}_{\bullet} = \sum_{j=1}^{J} c_j \overline{X}_{\bullet j} \qquad (1.6)$$

Variance rule for linear combinations

$$S_Y^2 = \sum_{j=1}^{J} c_j^2 S_j^2 + 2\sum_{j=2}^{J}\sum_{k=1}^{j-1} c_j c_k S_{jk} \qquad (1.7)$$

or, equivalently

$$S_Y^2 = \sum_{j=1}^{J}\sum_{k=1}^{J} c_j c_k S_{jk} \qquad (1.8)$$

Covariance rule for linear combinations

If Y is defined as in Equation (1.5), and W is a second linear combination, defined with linear weights d_k as

$$W_i = \sum_{j=1}^{J} d_j X_{ij} \tag{1.9}$$

then the covariance between W and Y is

$$S_{WY} = \sum_{j=1}^{J} \sum_{k=1}^{J} c_j d_k S_{jk} \tag{1.10}$$

The above rules can be equivalently expressed as "heuristic rules."

Write the linear combination as a rule for *variables*.

For example, if the LC W simply sums the X and Y scores, the variance of W can be obtained by first writing the expression for W as a sum of variables, i.e.,

$$X + Y \tag{1.11}$$

Algebraically square the above expression

$$(X + Y)^2 = X^2 + Y^2 + 2XY \tag{1.12}$$

Apply a "conversion rule". (1) Constants are left unchanged. (2) Squared variables become the variance of the variable. (3) Products of two variables become the covariance of the two variables.

The conversion rule, applied to Equation (1.12), yields

$$S_X^2 + S_Y^2 + 2S_{XY} \tag{1.13}$$

A similar rule holds for the covariance of two linear combinations.

Suppose we have

$$L = X + Y,\ M = X - Y \qquad (1.14)$$

The covariance between *L* and *M* is obtained by taking the *product* of the algebraic expressions for *L* and *M*, then applying the conversion rule.

We obtain

$$S_{LM} = S_X^2 - S_Y^2 \qquad (1.15)$$

Special Case – Uncorrelated Variables

$$S_Y^2 = \sum_{j=1}^{J} c_j^2 S_j^2 \qquad (1.16)$$

t Tests for Contrasts

A *contrast* is a linear combination with the linear weights summing to zero, i.e.,

$$\sum_{j=1}^{J} c_j = 0 \qquad (1.17)$$

In Psychology 310, we saw how a number of interesting hypotheses can be expressed in terms of *contrasts of population means*.

Often, the statistical null hypothesis is of the form

$$H_0 : \kappa = \sum_{j=1}^{J} c_j \mu_j = 0 \qquad (1.18)$$

If the J means are estimated from independent samples of size N_j, then a t statistic can be constructed as

$$t_{N_\bullet - J} = \frac{\sum_{j=1}^{J} c_j \overline{X}_{\bullet J}}{\sqrt{MS_e \sum_{j=1}^{J} \frac{c_j^2}{N_j}}} \qquad (1.19)$$

Orthogonal Contrasts

Two contrasts are *orthogonal* if the sum of cross-products of the linear weights is zero, i.e., if c_{ik} are weights for the *i*th contrast applied to the *k*th mean, then

$$\sum_{k=1}^{K} c_{ik} c_{jk} = 0, \text{for i} \neq \text{j} \qquad (1.20)$$

Basic ANOVA Designs

If you have taken an ANOVA course such as Psy 304B, this should be a just a quick review for you. If you haven’t taken an ANOVA course, you should at least try to grasp the basic idea behind several basic designs.

1-Way fixed and random effects.
2-Way.
1-Way Repeated Measures.
2-Way Between-Within.