Treating Time More Flexibly

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GCM, 2010
Treating Time More Flexibly

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Introduction

Our introductory examples have shared some simplifying features. Each is:

1. **Balanced.** Each individual is assessed an equal number of times.
2. **Time-Structured.** Each set of occasions is identical across individuals.

Moreover, we have used only:

1. **Time-Invariant Predictors.**
2. **A Standard Time Representation** which led to an easy interpretation of parameters.
The multilevel change model can handle more ambitious examples, where the data are not necessarily either balanced or time-structured. Moreover, we can include time-varying predictors.

Singer and Willett begin their Chapter 5 with a discussion of the difficulties of obtaining time-structured and balanced data in the real world.
Psychological Consequences of Unemployment

Example (Psychological Consequences of Unemployment)

- Ginexi, Howe, and Caplan (2000) designed a time-structured study with interviews scheduled a 1, 5, and 11 months after job loss.
- Once in the field, however, the interview times varied considerably around these targets, with increasing variability as the study proceeded.
- First interview (2–61 days), Second interview (111–220 days), Third interview (319–458 days)
- Ginexi et al. argued that number of days rather than target time should be used.
- As a result, data were not time-structured.
Accelerated Cohort Design

Example (Accelerated Cohort Design)

- Age-heterogeneous group is followed for a constant period of time
- Age is the appropriate time measure
- Different people are interviewed at different ages, for example
  - 14.2 → 15.2 → 16.2
  - 15.7 → 16.7 → 17.7
The CNLSY Study

Singer and Willett illustrate the structure of variably spaced data with an example from the Children of the National Longitudinal Study of Youth (CNLSY).

- The study assessed 3 waves of data on 89 African-American kids
- Ages 6.5, 8.5, 10.5.
- Outcome variable was the reading subtest of the Peabody Individual Achievement Test (PIAT)
- Actual times of measurement were unstructured.

We’ll jump to their slide set for a discussion of the example, then return for an analysis in R.
The CNLSY Study – AGE Model

```r
> data <- read.table("reading_pp.txt",header=T,sep="","")
> attach(data)
> library(lme4)
> age_c <- age - 6.5
> agegrp_c <- agegrp - 6.5
> fit.age <- lmer(piat ~ age_c + (1+age_c|id),REML=FALSE)
> fit.age

Linear mixed model fit by maximum likelihood
Formula: piat ~ age_c + (1 + age_c | id)
   AIC   BIC logLik deviance REMLdev
1816 1837    -902    1804    1804
Random effects:
 Groups     Name       Variance  Std.Dev.  Corr
    id (Intercept)  5.110      2.26   
age_c          3.300      1.82 0.576
Residual      27.452      5.24
Number of obs: 267, groups: id, 89

Fixed effects:
     Estimate Std. Error t value
(Intercept)   21.061    0.559    37.7
age_c          4.540    0.261    17.4

Correlation of Fixed Effects:
     (Intr) age_c
(Intr)   -0.287
age_c     
```

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The CNLSY Study – AGEGRP Model

```r
> fit.agegrp <- lmer(piat ~ agegrp_c + (1+agegrp_c|id),REML=FALSE)
> fit.agegrp

Linear mixed model fit by maximum likelihood
Formula: piat ~ agegrp_c + (1 + agegrp_c | id)
   AIC  BIC logLik deviance
1832 1853  -910  1820 1820
Random effects:
  Groups   Name          Variance  Std.Dev.  Corr
     id   (Intercept)  11.0       3.32       
     id    agegrp_c    4.4        2.10   0.236
     Residual        27.0       5.20       
Number of obs: 267, groups: id, 89

Fixed effects:
  Estimate  Std. Error     t value
(Intercept)  21.163   0.614       34.5
agegrp_c     5.031    0.296       17.0

Correlation of Fixed Effects:
   (Intr)
agegrp_c  -0.316
```

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The NLSY Wages Study – Model A

This is an unconditional growth model.

```
> detach(data)
> data <- read.table("wages_pp.txt",header=T,sep="","")
> attach(data)
> hgc_9 <- hgc - 9
> fit.A <- lmer(lnw ~ exper + (1 + exper | id), REML=FALSE)
> fit.A
```

Linear mixed model fit by maximum likelihood

Formula: lnw ~ exper + (1 + exper | id)

AIC  BIC  logLik deviance REMLdev
4933 4974  -2461  4921  4939

Random effects:

Groups   Name     Variance Std.Dev.  Corr
id  (Intercept) 0.05427   0.2330
    exper      0.00173   0.0415  -0.565
Residual                             0.09510  0.3084

Number of obs: 6402, groups: id, 888

Fixed effects:

Estimate Std. Error t value
(Intercept) 1.71560   0.01080  158.9
exper  0.04568   0.00234   19.5

Correlation of Fixed Effects:

  (Intr) exper  -0.565
```
The NLSY Wages Study – Model B

This model uses black and hgc_9 to predict slopes and intercepts of the individual's trajectory.

```
> fit.B <- lmer(lnw~exper+black+hgc_9+black:exper +hgc_9:exper + (1+exper|id),REML=FALSE)
> fit.B
```

## Linear mixed model fit by maximum likelihood
## Formula: lnw ~ exper + black + hgc_9 + black:exper + hgc_9:exper + (1 + exper | id)

<table>
<thead>
<tr>
<th>AIC</th>
<th>BIC</th>
<th>logLik</th>
<th>deviance</th>
<th>REMLdev</th>
</tr>
</thead>
<tbody>
<tr>
<td>4894</td>
<td>4961</td>
<td>-2437</td>
<td>4874</td>
<td>4925</td>
</tr>
</tbody>
</table>

### Random effects:

<table>
<thead>
<tr>
<th>Groups</th>
<th>Name</th>
<th>Variance</th>
<th>Std.Dev.</th>
<th>Corr</th>
</tr>
</thead>
<tbody>
<tr>
<td>id</td>
<td>(Intercept)</td>
<td>0.05175</td>
<td>0.2275</td>
<td></td>
</tr>
<tr>
<td>id</td>
<td>exper</td>
<td>0.00164</td>
<td>0.0404</td>
<td>-0.310</td>
</tr>
<tr>
<td>id</td>
<td>Residual</td>
<td>0.09519</td>
<td>0.3085</td>
<td></td>
</tr>
</tbody>
</table>

Number of obs: 6402, groups: id, 888

### Fixed effects:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>1.71714</td>
<td>0.01254</td>
<td>136.9</td>
</tr>
<tr>
<td>exper</td>
<td>0.04934</td>
<td>0.00263</td>
<td>18.7</td>
</tr>
<tr>
<td>black</td>
<td>0.01540</td>
<td>0.02393</td>
<td>0.6</td>
</tr>
<tr>
<td>hgc_9</td>
<td>0.03492</td>
<td>0.00788</td>
<td>4.4</td>
</tr>
<tr>
<td>exper:black</td>
<td>-0.01821</td>
<td>0.00850</td>
<td>-3.3</td>
</tr>
<tr>
<td>exper:hgc_9</td>
<td>0.00128</td>
<td>0.00172</td>
<td>0.7</td>
</tr>
</tbody>
</table>

### Correlation of Fixed Effects:

<table>
<thead>
<tr>
<th>Factor</th>
<th>(Intr)</th>
<th>exper</th>
<th>black</th>
<th>hgc_9</th>
<th>exper:black</th>
<th>exper:hgc_9</th>
</tr>
</thead>
<tbody>
<tr>
<td>exper</td>
<td>-0.575</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>black</td>
<td>-0.523</td>
<td>-0.301</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hgc_9</td>
<td>0.071</td>
<td>-0.020</td>
<td>-0.020</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exper:black</td>
<td>0.275</td>
<td>-0.478</td>
<td>-0.573</td>
<td>0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exper:hgc_9</td>
<td>-0.019</td>
<td>-0.003</td>
<td>0.011</td>
<td>-0.578</td>
<td>-0.023</td>
<td></td>
</tr>
</tbody>
</table>
The NLSY Wages Study – Model C

This “pared-back” model uses `black` to predict only the
intercepts and `hgc_9` to predict only the slopes of the
individual’s trajectory.

```r
> fit.C <- lmer(lnw~exper+hgc_9+black:exper + (1+exper|id),REML=FALSE)
> fit.C

Linear mixed model fit by maximum likelihood
Formula: lnw ~ exper + hgc_9 + black:exper + (1 + exper | id)
AIC  BIC logLik deviance REMLdev
4891  4945  -2437   4875   4910
Random effects:
Groups   Name        Variance Std.Dev.  Corr
id   (Intercept)  0.05183  0.2277
      exper       0.00165  0.0406 -0.312
Residual                 0.09517  0.3085
Number of obs: 6402, groups: id, 888

Fixed effects:
            Estimate Std. Error t value
(Intercept)  1.72147    0.01070   160.9
exper        0.04885    0.00251    19.4
hgc_9        0.03836    0.00643     6.0
exper:black  -0.01612    0.00451   -3.6

Correlation of Fixed Effects:
                      (Intr)  exper hgc_9
exper          -0.515
hgc_9          -0.515  0.077
exper:black   -0.036 -0.391 -0.015
```

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To demonstrate convergence problems, Model C was also fit to a reduced data set.

```r
> detach(data)
> data <- read.table("wages_small_pp.txt",header=T,sep="","
> attach(data)
> fit.C.small <- lmer(lnw~exper+hcg.9+black:exper + (1+exper|id),REML=FALSE)
> fit.C.small

Linear mixed model fit by maximum likelihood
Formula: lnw ~ exper + hcg.9 + black:exper + (1 + exper | id)
AIC BIC logLik deviance REMLdev
 300 328  -142  284   305
Random effects:
 Groups   Name   Variance Std.Dev. Corr
      id (Intercept) 8.22e-02 0.28662
             exper  3.52e-06 0.00188 1.000
Residual   1.15e-01 0.33907
Number of obs: 257, groups: id, 124

Fixed effects:
                        Estimate Std. Error t value
(Intercept)            1.7373    0.0476  36.5exper         0.0516    0.0211  2.4hcg.9         0.0461    0.0245  1.9exper:black -0.0597    0.0348 -1.7

Correlation of Fixed Effects:
    (Intr) exper  hcg.9
exper  -0.612
hcg.9   0.051 -0.133
exper:black -0.129 -0.297  0.023
```
Models for Missing Data

Certain kinds of missing data can be handled effectively by special methods. Some of the key *Random Component Selection Models* models for missing data include:

1. Missing Completely at Random (MCAR)
2. Covariate Dependent Dropout (CDD)
3. Missing at Random (MAR)
Missing Completely at Random

Suppose we denote the potential outcome variable by $y_i$, the random effect coefficients by $b_i$, and the covariates as $X_i$. The missingness mechanism is modeled as a random process $R_i$. When data are missing completely at random (MCAR), then

$$[R_i \mid X_i, y_i, b_i] = [R_i]$$

That is, the missingness mechanism is independent of the covariates, the outcome, and the random coefficients or, in other words, completely random.
Covariate Dependent Dropout

When data show covariate dependent dropout (CDD), we have

$$[R_i | X_i, y_i, b_i] = [R_i | X_i]$$ (2)

That is, the missingness mechanism is independent of the outcome and the random coefficients given the covariates. This model allows dependence of drop-out on both between-subject and within-subject covariates that can be treated as fixed in the model.
Missing at Random

Data are *Missing at Random* (MAR) if the distribution of the dropout mechanism depends on $y_i$ only through its observed components $y_{obs,i}$. That is

$$[R_i | X_i, y_{obs,i}, y_{mis,i}, b_i] = [R_i | X_i, y_{obs,i}]$$  \hspace{1cm} (3)
What to Do?

If a reasonable case can be made that the missing data mechanism is MCAR, CDD, or MAR, then ML methods applied to all the data will work well. However, if missingness depends on the random coefficients themselves or on the unobserved values in a way that cannot be predicted from covariates, then special approaches may be necessary.

This is a complex topic, probably worthy of a course in itself. The books by Joe Shafer and Little and Rubin, and the 1995 JASA article (vol 90, pp. 1112–1121, available online) are primary references.
What to Do?

A MCAR test is available, and rejecting the null hypothesis rejects the MCAR assumption. However, since the goal is *not* to reject, the standard caveats about Accept-Support testing apply.

If missingness is clearly non-ignorable, you need to either model the mechanism or use a pattern mixture model.
Time-Varying Predictors

Time-varying predictors can change values at any recording instance.

Fortunately, the person-period data format handles such data effortlessly.
The Ginexi et al. Unemployment Study

This study examined the relationship over time between unemployment and depression.

```r
> detach(data)
> data <- read.table("unemployment_pp.txt", + header=T,sep="",")
> attach(data)
```

(Jump to Singer-Willett Chapter 5 slide set.)
Model A – An Unconditional Growth Model

\[ Y_{ij} = \pi_0 i + \pi_1 i \, TIME_{ij} + \epsilon_{ij} \]

with

\[
\begin{align*}
\pi_0 i &= \gamma_0 + \zeta_0 i \\
\pi_1 i &= \gamma_1 + \zeta_1 i
\end{align*}
\]

and the standard assumption. Substituting, we get the model

\[ Y_{ij} = \gamma_0 + \gamma_1 \, TIME_{ij} + \zeta_0 i + \zeta_1 \, TIME_{ij} + \epsilon_{ij} \]
Fitting Model A

```r
> fit.A <- lmer(cesd ~ 1 + months +
  + (1+months|id),REML=FALSE)
> fit.A

Linear mixed model fit by maximum likelihood
Formula: cesd ~ 1 + months + (1 + months | id)
   AIC  BIC logLik deviance
5145 5172  -2567  5133
Random effects:
  Groups   Name        Variance  Std.Dev.  Corr
     id  (Intercept)  86.848     9.319
         months     0.355     0.596 -0.551
     Residual     68.850     8.298
Number of obs: 674, groups: id, 254

Fixed effects:
             Estimate Std. Error t value
(Intercept)  17.669      0.776  22.78
months      -0.422      0.083  -5.09

Correlation of Fixed Effects:
              (Intr) months
(Intercept) 1.000 -0.632
months      -0.632 1.000
```

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Model B – Adding Unemployment as a Time-Varying Predictor

Next, unemployment is added as a direct level-1 predictor, yielding the composite model

\[ Y_{ij} = \gamma_0 + \gamma_{10} TIME_{ij} + \gamma_{20} UNEMP_{ij} + \zeta_0 + \zeta_{1i} TIME_{ij} + \epsilon_{ij} \]
Fitting Model B

```r
> fit.B <- lmer(cesd ~ 1 + months +
+ unemp + (1+months|id),REML=FALSE)
> fit.B

Linear mixed model fit by maximum likelihood
Formula: cesd ~ 1 + months + unemp + (1 + months | id)
   AIC  BIC logLik deviance   df.resid
5122 5153  -2554   5108      5108
Random effects:
 Groups Name          Variance Std.Dev. Corr
     id  (Intercept) 93.519    9.671
       months  0.465    0.682   -0.591
    Residual    62.388   7.899
Number of obs: 674, groups: id, 254

Fixed effects:
 (Intercept)   months  unemp
     12.6656    -0.2020     5.1113

Correlation of Fixed Effects:
   (Intr) months
months  -0.715
       unemp -0.780
```

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Model C – Allowing the Effect of Unemployment to Vary over Time

Next, the effect of unemployment is allowed to change over time via the addition of an interaction term.

\[ Y_{ij} = \gamma_{00} + \gamma_{10} TIME_{ij} + \gamma_{20} UNEMP_{ij} + \gamma_{30} UNEMP_{ij} \times TIME_{ij} + \zeta_0 + \zeta_1 TIME_{ij} + \epsilon_{ij} \]
### Fitting Model C

```r
> fit.C <- lmer(cesd ~ 1 + months + unemp + months:unemp + (1+months|id),REML=FALSE)
> fit.C

Linear mixed model fit by maximum likelihood
Formula: cesd ~ 1 + months + unemp + months:unemp + (1 + months | id)
AIC  BIC  logLik deviance REMLdev
5119 5155 -2552    5103 5105

Random effects:
Groups   Name   Variance Std.Dev. Corr
id       (Intercept) 93.713  9.681
         months    0.451  0.672 -0.596
Residual         62.031  7.876
Number of obs: 674, groups: id, 254

Fixed effects:
             Estimate Std. Error t value
(Intercept)   9.617      1.889    5.09
months        0.162      0.194    0.84
unemp         8.529      1.878    4.54
months:unemp  -0.465      0.217   -2.14

Correlation of Fixed Effects:
        (Intr) months unemp
months -0.888
unemp  -0.911  0.863
months:unmp  0.755 -0.878 -0.852
```
In this model, the trajectory is constrained to have a zero slope when individuals are employed.

This is done by including both a main effect for unemployment and an interaction effect between unemployment and time at both the fixed and random levels, and removing the fixed and random effects for time.

Since unemployment is a binary variable, the net effect of this is that when unemployment is 1, the interaction effect solely determines the slope of the relationship between $Y$ and time. When unemployment is zero, there is no slope term, and so the slope effectively becomes zero.

$$ Y_{ij} = \gamma_{00} + \gamma_{20} \text{UNEMP}_{ij} + \gamma_{30} \text{UNEMP}_{ij} \times \text{TIME}_{ij} + \zeta_{0i} + \zeta_{2i} \text{UNEMP}_{ij} + \zeta_{3i} \text{UNEMP}_{ij} \times \text{TIME}_{ij} + \epsilon_{ij} $$
Fitting Model C

```r
> fit.D <- lmer(cesd ~ 1 + unemp +
+ months:unemp + (1+unemp + months:unemp|id),REML=FALSE)
> fit.D

Linear mixed model fit by maximum likelihood
Formula: cesd ~ 1 + unemp + months:unemp + (1 + unemp + months:unemp | id)
   AIC  BIC logLik deviance REMLdev
5115 5160  -2548  5095     5096
Random effects:
Groups   Name        Variance Std.Dev. Corr
id   (Intercept)   45.254    6.727
     unemp         44.968    6.706   0.145
     unemp:months  0.753     0.868  0.112 -0.967
Residual              59.018    7.682
Number of obs: 674, groups: id, 254

Fixed effects:
             Estimate Std. Error  t value
(Intercept)  11.195      0.790   14.17
unemp        6.927      0.930    7.45
unemp:months -0.303      0.112  -2.70

Correlation of Fixed Effects:
   (Intr) unemp
unemp   -0.563
unemp:mnths -0.074 -0.443
```

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Recentering the Effects of Time

So far, time has been centered on the initial status point.

However, other alternatives are possible, and any meaningful constant can be used.

Singer and Willett discuss some options in the context of a study by Tomarken, et al. (1997).
The composite model is

\[ Y_{ij} = \gamma_{00} + \gamma_{01} TREAT_i + \gamma_{10} TIME_{ij} \\
+ \gamma_{11} TREAT_i \times TIME_{ij} + \epsilon_{ij} + (\zeta_{1i} TIME_{ij} + \zeta_{0i}) \]
Fitting the Model

```r
> detach(data)
> data <- read.table("medication_pp.txt",header=T,sep="","
> attach(data)
> fit.initial <- lmer(pos ~ treat + time + treat:time + (1 + time | id), REML=FALSE)
> fit.initial

Linear mixed model fit by maximum likelihood
Formula: pos ~ treat + time + treat:time + (1 + time | id)
AIC  BIC logLik deviance REMLdev
12696 12737 -6340 12680 12663
Random effects:
 Groups     Name        Variance  Std.Dev.  Corr
 id (Intercept) 2111.4     45.95     -
       time       63.7      7.98  -0.332
 Residual      1229.9     35.07
Number of obs: 1242, groups: id, 64

Fixed effects:
            Estimate Std. Error t value
(Intercept) 167.46     9.33      17.96
 treat      -3.11     12.33     -0.25
 time       -2.42      1.73     -1.40
 treat:time  5.54      2.28      2.43

Correlation of Fixed Effects:
            (Intr) treat  time
 treat  -0.756
 time   -0.404  0.305
 treat:time  0.307 -0.408 -0.760
```

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Fitting the Model Centered at Midpoint

```r
> fit.midpoint <- lmer(pos ~ treat + time333 + treat:time333 + (1 + time333 | id), REML=FALSE)
> fit.midpoint

Linear mixed model fit by maximum likelihood
Formula: pos ~ treat + time333 + treat:time333 + (1 + time333 | id)
   AIC  BIC logLik deviance REMLdev
12696 12737 -6340 12680 12663
Random effects:
 Groups Name Variance Std.Dev. Corr
 id   (Intercept) 2008.8  44.82 
      time333  63.7   7.98   0.254
 Residual  1229.9  35.07
Number of obs: 1242, groups: id, 64

Fixed effects:
             Estimate Std. Error t value
(Intercept) 159.40      8.76  18.19
  treat     15.35     11.54   1.33
 time333  -2.42      1.73  -1.40
treat:time333  5.54      2.28   2.43

Correlation of Fixed Effects:
             (Intr) treat tim333
 treat     -0.759
 time333   0.229 -0.173
treat:tim333 -0.174  0.221 -0.760
```
Fitting the Model Centered at Endpoint

```r
> fit.endpoint <- lmer(pos ~ treat + time667 + treat:time667 + (1 + time667 | id), REML=FALSE)
> fit.endpoint

Linear mixed model fit by maximum likelihood
Formula: pos ~ treat + time667 + treat:time667 + (1 + time667 | id)
   AIC   BIC logLik deviance REMLdev
12696 12737   -6340     12680 12663
Random effects:
Groups   Name        Variance   Std.Dev.   Corr
    id     (Intercept) 3322.5  57.64 time667
time667  63.7      7.98     -0.512
   Residual       1229.9  35.07
Number of obs: 1242, groups: id, 64

Fixed effects:
   Estimate Std. Error t value  
(Intercept)   151.34      11.54   13.11
  treat        33.80      15.16    2.23
time667     -2.42       1.73   -1.40
treat:time667  5.54       2.28    2.43

Correlation of Fixed Effects:
   (Intr)     treat    time667     time667 treat:tm667
(Intercept) -0.761
  treat     -0.512
 time667      0.673
 treat:time667 0.670
```