Introduction to Set Theory for Statistics
Class Notes
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Definition. A *Set* is any well defined collection of "objects."

Definition. The *elements* of a set are the objects in a set.

Notation. Usually we denote sets with upper-case letters, elements with lower-case letters. The following notation is used to show set membership

$x \in \mathrm{A}$ means that x is a member of the set A

$x \notin \mathrm{A}$ means that x is not a member of the set A.

Ways of describing sets.

List the elements. $\mathrm{A} = \{1,2,3,4,5,6\}$

Give a *Verbal Description* "A is the set of all integers from 1 to 6, inclusive."

Give a *Mathematical Inclusion Rule* $\mathrm{A} = \{x \mid 0 \le x \le 1\}$

Some Special Sets

$\varnothing$ the "Null Set," or "Empty Set," a set with no elements.

Ω the "Universal Set," a set containing all the elements currently under consideration."

Set Theory Ideas

Definition. Subset.

$\mathrm{A} \subseteq \mathrm{B}$ "A is a subset of B"

We say "A is a subset of B" if $x \in \mathrm{A} \Rightarrow x \in \mathrm{B}$, i.e., all the members of A are also members of B. The notation for subset is very similar to the notation for "less than or equal to," and means, in terms of the sets, "included in or equal to."

Definition. Proper Subset.

$\mathrm{A} \subset \mathrm{B}$ "A is a proper subset of B"

We say "A is a proper subset of B" if $x \in \mathrm{A} \Rightarrow x \in \mathrm{B}$, i.e., all the members of A are also members of B, but in addition there exists at least one element c such that $c \in \mathrm{B}$ but $c \notin \mathrm{A}$. The notation for subset is very similar to the notation for "less than," and means, in terms of the sets, "included in but not equal to."

Definition. Union of two sets. $\mathrm{A} \cup \mathrm{B}$

The set "A union B" denoted $\mathrm{A} \cup \mathrm{B}$, is the set composed of the elements that are in A, or B, or both. This is similar to the logical "or" operator.

Definition. Intersection of two sets. $A \cap B$

The set "A intersect B", denoted $A \cap B$, is composed of the eements that are in both A and B, i.e., the elements that are common to both sets. Note the similarity of the set intersection operator to the logical "and."

Definition. Set complement. $\overline{A}$

The set "A complement," or "not A," denoted $\overline{A}$,is composed of those elements which are not in the set A (but are in the universal set Ω .

Definition. Set difference. $A - B$

The set difference "A minus B" is the set of elements that are in A, with those that are in B subtracted out. Another way of putting it is, it is the set of elements that are in A, *and* not in B.

Hence, we have, from set theory itself, another definition. Specifically. $A - B = A \cap \overline{B}$.

Examples.

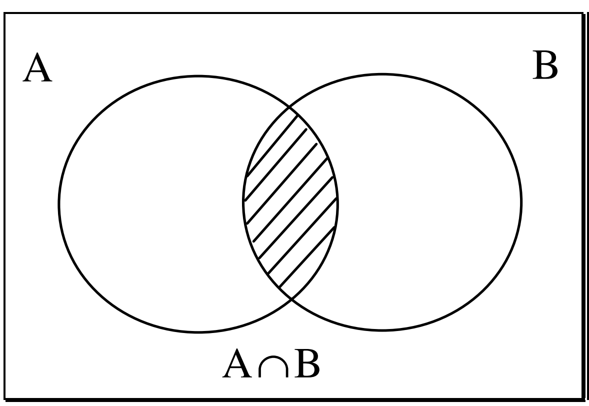
$A = \{1,2,3\}$ $B = \{3,4,5,6\}$

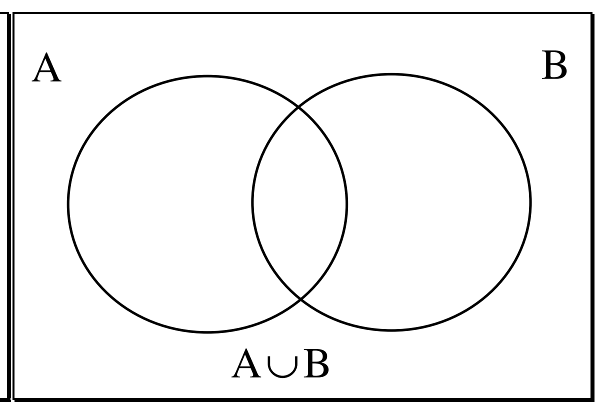
$A \cap B = \{3\}$ $A \cup B = \{1,2,3,4,5,6\}$ $B - A = \{4,5,6\}$

Venn Diagrams.

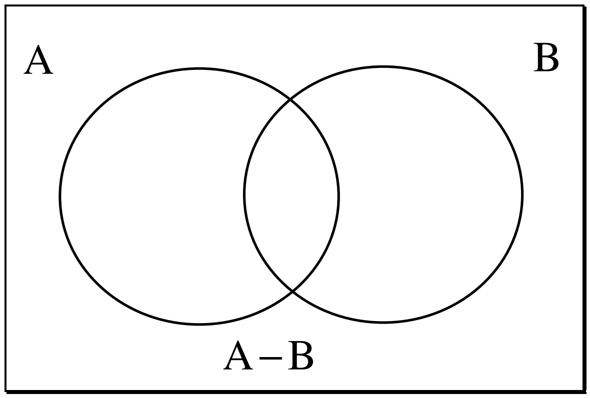
Venn Diagrams use topological areas to stand for sets.

Examples. I've done this one for you. Try this one for yourself!





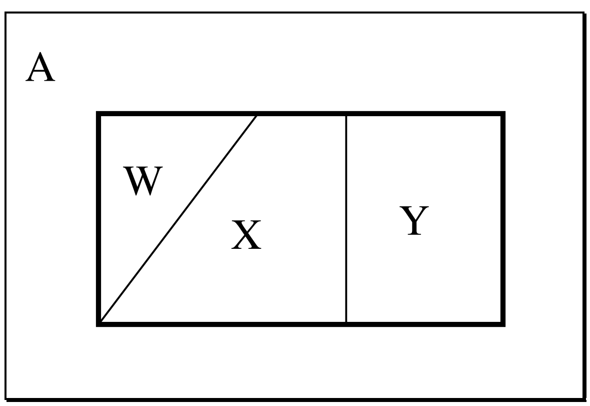
Give this one a try as well. Shade in the designated area.



Definition. We say that a group of sets is *exhaustive* of another set if their union is equal to that set. For example, if $C = A \cup B$, we say that A and B are exhaustive with respect to C.

Definition. We say that two sets A and B are *mutually exclusive* if $A \cap B = \varnothing$, that is, the sets have no elements in common.

Definition. We say that a group of sets *partitions* another set if they are mutually exclusive and exhaustive with respect to that set. When we "partition a set," we break it down into mutually exclusive and exhaustive regions, i.e., regions with no overlap. The Venn diagram below should help you get the picture. In this diagram, the set A (the rectangle) is partitioned into sets W,X, and Y.



Examples. To test your understanding of the above concepts, try to figure out for yourself the following universal facts about sets. Pencil in your answers below each question.

1. $A \cup \varnothing = ?$ 2. $A \cap \varnothing = ?$ 3. $A \cup \overline{A} = ?$ 4. $A - \overline{A} = ?$ 5. $A \cap \overline{A} = ?$

6. $A \cup \Omega = ?$ 7. $A \cap \Omega = ?$ 8. If $A \subset B$, then $A \cap B = ?$ 9. $A \subset B$, then $A \cup B = ?$